MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE MAIN-2021

COMPUTER BASED TEST (CBT)

DATE : 17-03-2021 (MORNING SHIFT) | TIME : (9.00 am to 12.00 pm)

Duration 3 Hours | Max. Marks : 300

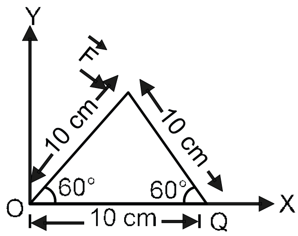
QUESTION & SOLUTIONS

PART A : PHYSICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. A triangular plate is shown. A force $\vec{F} = 4\hat{i} - 3\hat{j}$ is applied at point P. The torque at point P with respect to point 'O' and 'Q' are :



(1) $-15 - 20\sqrt{3}, 15 - 20\sqrt{3}$ (2) $15 + 20\sqrt{3}, 15 - 20\sqrt{3}$

(3) $15 - 20\sqrt{3}, 15 + 20\sqrt{3}$ (4) $-15 + 20\sqrt{3}, 15 + 20\sqrt{3}$

Ans. (1)

Sol. $\vec{F} = 4\hat{i} - 3\hat{j}$

$\vec{r}_1 = 5\hat{i} + 5\sqrt{3}\hat{j}$ & $\vec{r}_2 = -5\hat{i} + 5\sqrt{3}\hat{j}$

Torque about 'O'

$\vec{\tau}_O = \vec{r}_1 \times F = \left(-15 - 20\sqrt{3}\right)\hat{k} = \left(15 + 20\sqrt{3}\right)\left(-\hat{k}\right)$

Torque about 'Q'

$\vec{\tau}_Q = \vec{r}_2 \times F = \left(-15 + 20\sqrt{3}\right)\hat{k} = \left(15 - 20\sqrt{3}\right)\left(-\hat{k}\right)$

2. When two soap bubbles of radii a and b (b > a) coalesce, the radius of curvature of common surface is:

(1) $\frac{ab}{b-a}$ (2) $\frac{a+b}{ab}$ (3) $\frac{b-a}{ab}$ (4) $\frac{ab}{a+b}$

Ans. (1)

Sol. Excess pressure at common surface is given by

$P_{ex} = 4T\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{4T}{r}$

$\therefore \frac{1}{r} = \frac{1}{a} - \frac{1}{b}$

$r = \frac{ab}{b-a}$

3. A polyatomic ideal gas has 24 vibrational modes. What is the value of γ ?

(1) 1.03 (2) 1.30 (3) 1.37 (4) 10.3

Ans. (1)

Sol. Since each vibrational mode has 2 degrees of freedom hence total vibrational degrees of freedom = 48

f = 3 + 3 + 48 = 54

$$\gamma = 1 + \frac{2}{f} = \frac{28}{27} = 1.03$$

4. If an electron is moving in the n^{th} orbit of the hydrogen atom, then its velocity (v_n) for the n^{th} orbit is given as :

(1) $v_n \propto n$ (2) $v_n \propto \frac{1}{n}$ (3) $v_n \propto n^2$ (4) $v_n \propto \frac{1}{n^2}$

Ans. (2)

Sol. We know velocity of electron in n^{th} shell of hydrogen atom is given by

$$v = \frac{2\pi kZe^2}{nh}$$

$$\therefore v \propto \frac{1}{n}$$

5. An electron of mass m and a photon have same energy E. The ratio of wavelength of electron to that of photon is : (c being the velocity of light)

(1) $\frac{1}{c}\left(\frac{2m}{E}\right)^{1/2}$ (2) $\frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$ (3) $\left(\frac{E}{2m}\right)^{1/2}$ (4) $c\,(2mE)^{1/2}$

Ans. (2)

Sol. $$\lambda_1 = \frac{h}{\sqrt{2mE}}$$

$$\lambda_2 = \frac{hc}{E}$$

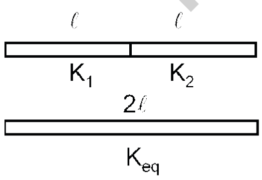
$$\frac{\lambda_1}{\lambda_2} = \frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$$

6. Two identical metal wires of thermal conductivities K_1 and K_2 respectively are connected in series. The effective thermal conductivity of the combination is :

(1) $\frac{2K_1K_2}{K_1+K_2}$ (2) $\frac{K_1+K_2}{2K_1K_2}$ (3) $\frac{K_1+K_2}{K_1K_2}$ (4) $\frac{K_1K_2}{K_1+K_2}$

Ans. (1)

Sol.



$$R_{eff} = \frac{\ell}{K_1A} + \frac{\ell}{K_2A} = \frac{2\ell}{K_{eq}A}$$

$$K_{eq} = \frac{2K_1K_2}{K_1 + K_2}$$

7. The vernier scale used for measurement has a positive zero error of 0.2 mm. If while taking a measurement it was noted that '0' on the vernier scale lies between 8.5 cm and 8.6 cm, vernier coincidence is 6, then the correct value of measurement is_____ cm.

(least count = 0.01 cm)

(1) 8.36 cm (2) 8.54 cm (3) 8.58 cm (4) 8.56 cm

Ans. (2)

Sol. Positive zero error = 0.2 mm

Main scale reading = 8.5 cm

Vernier scale reading = 6 × 0.01 = 0.06 cm

Final reading = 8.5 + 0.06 – 0.02 = 8.54 cm

8. An AC current is given by $I = I_1 \sin\omega t + I_2 \cos\omega t$. A hot wire ammeter will give a reading :

(1) $\sqrt{\frac{I_1^2 - I_2^2}{2}}$ (2) $\sqrt{\frac{I_1^2 + I_2^2}{2}}$ (3) $\frac{I_1 + I_2}{\sqrt{2}}$ (4) $\frac{I_1 + I_2}{2\sqrt{2}}$

Ans. (2)

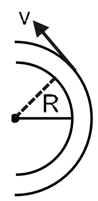
Sol. $I = I_1 \sin \omega t + I_2 \cos \omega t$

$$\therefore I_0 = \sqrt{I_1^2 + I_2^2}$$

$$\therefore I_{rms} = \frac{I_0}{\sqrt{2}} = \sqrt{\frac{I_1^2 + I_2^2}{2}}$$

9. A modern grand-prix racing car of mass m is travelling on a flat track in a circular arc of radius R with a speed v. If the coefficient of static friction between the tyres and the track is μ_s, then the magnitude of negative lift F_L acting downwards on the car is :

(Assume forces on the four tyres are identical and g = acceleration due to gravity)



(1) $m\left(\frac{v^2}{\mu_s R} + g\right)$ (2) $m\left(\frac{v^2}{\mu_s R} - g\right)$ (3) $m\left(g - \frac{v^2}{\mu_s R}\right)$ (4) $-m\left(g + \frac{v^2}{\mu_s R}\right)$

Ans. (2)

Sol. $\mu_s N = \frac{mv^2}{R}$

$$N = \frac{mv^2}{\mu_s R} = mg + F_L$$

$$F_L = \frac{mv^2}{\mu_s R} - mg$$

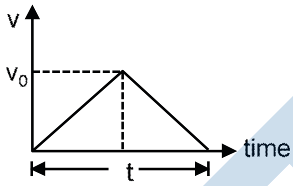
10. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t seconds, the total distance travelled is :

(1) $\frac{4\alpha\beta}{(\alpha+\beta)}t^2$ (2) $\frac{2\alpha\beta}{(\alpha+\beta)}t^2$ (3) $\frac{\alpha\beta}{2(\alpha+\beta)}t^2$ (4) $\frac{\alpha\beta}{4(\alpha+\beta)}t^2$

Ans. (3)

Sol. $v_0 = \alpha t_1$ and $0 = v_0 - \beta t_2 \Rightarrow v_0 = \beta t_2$

$t_1 + t_2 = t$



$$v_0\left(\frac{1}{\alpha}+\frac{1}{\beta}\right) = t$$

$$\Rightarrow v_0 = \frac{\alpha\beta t}{\alpha+\beta}$$

Distance = area of v-t graph

$$= \frac{1}{2}\times t \times v_0 = \frac{1}{2}\times t \times \frac{\alpha\beta t}{\alpha+\beta} = \frac{\alpha\beta t^2}{2(\alpha+\beta)}$$

11. A solenoid of 1000 turns per metre has a core with relative permeability 500. Insulated windings of the solenoid carry an electric current of 5A. The magnetic flux density produced by the solenoid is :

(permeability of free space = $4\pi \times 10^{-7}$ H/m)

(1) πT (2) $2 \times 10^{-3}\ \pi$T (3) $\frac{\pi}{5}$T (4) $10^{-4}\ \pi$T

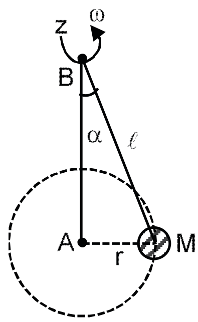
Ans. (1)

Sol. $B = \mu nI = \mu_0 m_r nI$

$B = 4\pi \times 10^{-7} \times 500 \times 1000 \times 5$

$B = \pi$ Tesla

12. A mass M hangs on a massless rod of length ℓ which rotates at a constant angular frequency. The mass M moves with steady speed in a circular path of constant radius. Assume that the system is in steady circular motion with constant angular velocity ω. The angular momentum of M about point A is L_A which lies in the positive z direction and the angular momentum of M about B is L_B. The correct statement for this system is :



(1) L_A and L_B are both constant in magnitude and direction

(2) L_B is constant in direction with varying magnitude

(3) L_B is constant, both in magnitude and direction

(4) L_A is constant, both in magnitude and direction

Ans. (4)

Sol. We know, $\vec{L} = m(\vec{r} \times \vec{v})$

Now with respect to A, we always get direction of $\vec{L}$ along +ve z-axis and also constant magnitude as mvr. But with respect to B, we get constant magnitude but continuously changing direction.

13. For what value of displacement the kinetic energy and potential energy of a simple harmonic oscillation become equal ?

(1) x = 0 (2) x = ± A (3) $x = \pm\frac{A}{\sqrt{2}}$ (4) $x = \frac{A}{2}$

Ans. (3)

Sol. KE = PE

$$\frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}m\omega^2 x^2$$

$$A^2 - x^2 = x^2$$

$$2x^2 = A^2$$

$$x = \pm\frac{A}{\sqrt{2}}$$

14. A Carnot's engine working between 400 K and 800 K has a work output of 1200 J per cycle. The amount of heat energy supplied to the engine from the source in each cycle is :

(1) 3200 J (2) 1800 J (3) 1600 J (4) 2400 J

Ans. (4)

Sol. $\eta = \frac{T_2}{T_1} = \frac{Q_2}{Q_1} = \frac{Q_1 - W}{Q_1}$ $(\because W = Q_1 - Q_2)$

$$\frac{400}{800} = 1 - \frac{W}{Q_1}$$

$$\frac{W}{Q_1} = 1 - \frac{1}{2} = \frac{1}{2}$$

$Q_1 = 2W = 2400$ J

15. The thickness at the centre of a plano convex lens is 3 mm and the diameter is 6 cm. If the speed of light in the material of the lens is 2×10^8 ms^{-1}. The focal length of the lens is _____.

(1) 0.30 cm (2) 15 cm (3) 1.5 cm (4) 30 cm

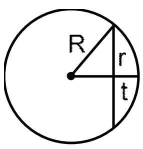
Ans. (4)

Sol. $R^2 = r^2 + (R - t)^2$

$R^2 = r^2 + R^2 + t^2 - 2Rt$

Neglecting t^2, we get

$$R = \frac{r^2}{2t}$$



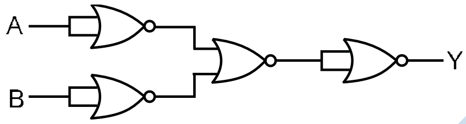
$$\therefore \frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{\mu - 1}{R}$$

$$f = \frac{R}{\mu - 1} = \frac{r^2}{2t(\mu - 1)} = \frac{(3 \times 10^{-2})^2}{2 \times 3 \times 10^{-3} \times \left(\frac{3}{2} - 1\right)}$$

$$= \frac{9 \times 10^{-4}}{6 \times 10^{-3} \times 1} \times 2$$

f = 0.3 m = 30 cm

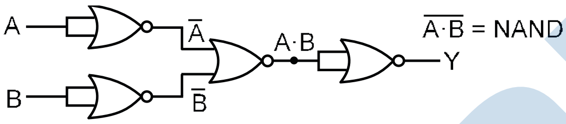
16. The output of the given combination gates represents :



(1) XOR Gate (2) NAND Gate (3) AND Gate (4) NOR Gate

Ans. (2)

Sol. By De Morgan's theorem, we have



17. A boy is rolling a 0.5 kg ball on the frictionless floor with the speed of 20 ms^{-1}. The ball gets deflected by an obstacle on the way. After deflection it moves with 5% of its initial kinetic energy. What is the speed of the ball now ?

(1) 19.0 ms^{-1} (2) 4.47 ms^{-1} (3) 14.41 ms^{-1} (4) 1.00 ms^{-1}

Ans. (2)

Sol. Given, m = 0.5 kg and u = 20 m/s

Initial kinetic energy $(k_i) = \frac{1}{2}mu^2$

$$= \frac{1}{2} \times 0.5 \times 20 \times 20 = 100 \text{ J}$$

After deflection it moves with 5% of k_i

$$\therefore k_f = \frac{5}{100} \times k_i \Rightarrow \frac{5}{100} \times 100$$

$\Rightarrow k_f = 5$ J

Now, let the final speed be 'v' m/s, then :

$$k_f = 5 = \frac{1}{2}mv^2$$

$\Rightarrow v^2 = 20$

$$\Rightarrow v = \sqrt{20} = 4.47 \text{ m/s}$$

18. Which level of the single ionized carbon has the same energy as the ground state energy of hydrogen atom?

(1) 1 (2) 6 (3) 4 (4) 8

Ans. (2)

Sol. Energy of H-atom is $E = -13.6\ Z^2/n^2$

for H-atom Z = 1 & for ground state, n = 1

$$\Rightarrow E = -13.6 \times \frac{1^2}{1^2} = -13.6 \text{ eV}$$

Now for carbon atom (single ionised), Z = 6

$$E = -13.6\frac{Z^2}{n^2} = -13.6 \quad \text{(given)}$$

$\Rightarrow n^2 = 6^2 \Rightarrow n = 6$

19. Two ideal polyatomic gases at temperatures T_1 and T_2 are mixed so that there is no loss of energy. If F_1 and F_2, m_1 and m_2, n_1 and n_2 be the degrees of freedom, masses, number of molecules of the first and second gas respectively, the temperature of mixture of these two gases is :

(1) $\frac{n_1T_1 + n_2T_2}{n_1 + n_2}$ (2) $\frac{n_1F_1T_1 + n_2F_2T_2}{n_1F_1 + n_2F_2}$ (3) $\frac{n_1F_1T_1 + n_2F_2T_2}{F_1 + F_2}$ (4) $\frac{n_1F_1T_1 + n_2F_2T_2}{n_1 + n_2}$

Ans. (2)

Sol. Let the final temperature of the mixture be T.

Since, there is no loss in energy.

$\Delta U = 0$

$$\Rightarrow \frac{F_1}{2}n_1R\ \Delta T + \frac{F_2}{2}n_2R\ \Delta T = 0$$

$$\Rightarrow \frac{F_1}{2}n_1R\ (T_1 - T) + \frac{F_2}{2}n_2R\ (T_2 - T) = 0$$

$$\Rightarrow T = \frac{F_1n_1RT_1 + F_2n_2RT_2}{F_1n_1R + F_2n_2R} \Rightarrow \frac{F_1n_1T_1 + F_2n_2T_2}{F_1n_1 + F_2n_2}$$

20. A current of 10A exists in a wire of cross-sectional area of 5 mm^2 with a drift velocity of $2 \times 10^{-3}\ ms^{-1}$. The number of free electrons in each cubic meter of the wire is ___.

(1) 2×10^6 (2) 625×10^{25} (3) 2×10^{25} (4) 1×10^{23}

Ans. (2)

Sol. $i = 10A,\ A = 5\ mm^2 = 5 \times 10^{-6}\ m^2$

and $v_d = 2 \times 10^{-3}$ m/s

We know, i = neAvd

$\therefore 10 = n \times 1.6 \times 10^{-19} \times 5 \times 10^{-6} \times 2 \times 10^{-3}$

$\Rightarrow n = 0.625 \times 10^{28} = 625 \times 10^{25}$

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. For VHF signal broadcasting, ____ km^2 of maximum service area will be covered by an antenna tower of height 30m, if the receiving antenna is placed at ground. Let radius of the earth be 6400 km.

(Take π as 3.14)

Ans. (1206)

Sol. $d = \sqrt{2Rh}$

$A = \pi d^2$

$A = \pi 2Rh$

$= 3.14 \times 2 \times 6400 \times \frac{30}{1000}$

$A = 1205.76\ km^2$

$A \simeq 1206\ km^2$

2. The angular speed of truck wheel is increased from 900 rpm to 2460 rpm in 26 seconds. The number of revolutions by the truck engine during this time is _______.

(Assuming the acceleration to be uniform).

Ans. (728)

Sol. We know, $\theta = \left(\frac{\omega_1 + \omega_2}{2}\right)t$

Let number of revolutions be N

$\therefore\ 2\pi N = 2\pi\left(\frac{900 + 2460}{60 \times 2}\right) \times 26$

N = 728

3. The equivalent resistance of series combination of two resistors is 's'. When they are connected in parallel, the equivalent resistance is 'p'. If s = np, then the minimum value for n is ___.

Ans. (4)

Sol. $R_1 + R_2 = s$ ... (1)

$\frac{R_1R_2}{R_1 + R_2} = p$ ... (2)

$R_1R_2 = sp$

$R_1R_2 = np^2$

$R_1 + R_2 = \frac{nR_1R_2}{(R_1 + R_2)}$

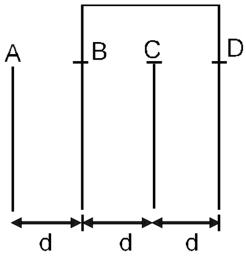
$\frac{(R_1 + R_2)^2}{R_1R_2} = n$

for minimum value of n

$R_1 = R_2 = R$

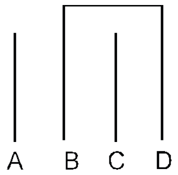
$\therefore n = \dfrac{(2R)^2}{R^2} = 4$

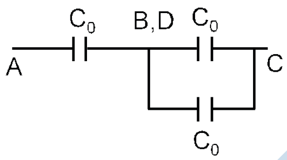
4. Four identical rectangular plates with length, $\ell = 2$ cm and breadth, $b = \dfrac{3}{2}$ cm are arranged as shown in figure. The equivalent capacitance between A and C is $\dfrac{x\,\varepsilon_0}{d}$. The value of x is ___.



Ans. (2)

Sol.





$C_{eq} = \dfrac{2C_0}{3} = \dfrac{2}{3}\dfrac{\epsilon_0 A}{d}$

$C_{eq} = \dfrac{2\epsilon_0}{3d} \times \left(2 \times \dfrac{3}{2}\right) = 2 \quad (\because A = \ell b = 2 \times \dfrac{3}{2})$

5. The radius in kilometer to which the present radius of earth (R = 6400 km) to be compressed so that the escape velocity is increased 10 time is _______.

Ans. (64)

Sol. $V_e = \sqrt{\dfrac{2Gm}{R}}$ .... (1)

$10V_e = \sqrt{\dfrac{2Gm}{R'}}$ ... (2)

$\therefore 10 = \sqrt{\dfrac{R}{R'}}$

$\Rightarrow R' = \dfrac{R}{100} = \dfrac{6400}{100} = 64$ km

6. Consider two identical springs each of spring constant k and negligible mass compared to the mass M as shown. Fig.1 shows one of them and Fig.2 shows their series combination. The ratios of time period of oscillation of the two SHM is $\frac{T_b}{T_a} = \sqrt{x}$, where value of x is ______.

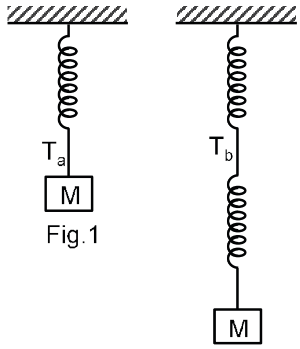


Fig.1 Fig.2

Ans. (2)

Sol. $T_a = 2\pi\sqrt{\frac{M}{K}}$

$T_b = 2\pi\sqrt{\frac{M}{K/2}}$

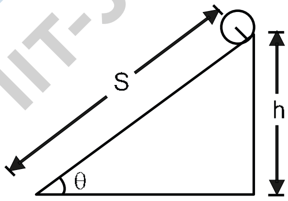
$\frac{T_b}{T_a} = \sqrt{2} = \sqrt{x} \qquad \Rightarrow x = 2$

7. The following bodies,

(1) a ring (2) a disc (3) a solid cylinder (4) a solid sphere,

of same mass 'm' and radius 'R' are allowed to roll down without slipping simultaneously from the top of the inclined plane. The body which will reach first at the bottom of the inclined plane is ______.

[Mark the body as per their respective numbering given in the question]



Ans. (4)

Sol. Mg sinθ R = (mk^2 + mR^2) α

$\alpha = \frac{Rg\sin\theta}{k^2 + R^2} \quad \Rightarrow \quad a = \frac{g\sin\theta}{1 + \frac{k^2}{R^2}}$

$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{g\sin\theta}\left(1 + \frac{k^2}{R^2}\right)}$

for least time, k should be least & we know k is least for solid sphere.

8. A parallel plate capacitor whose capacitance C is 14 pF is charged by a battery to a potential difference V = 12V between its plates. The charging battery is now disconnected and a porcelain plate with k = 7 is inserted between the plates, then the plate would oscillate back and forth between the plates with a constant mechanical energy of ______ pJ.

(Assume no friction)

Ans. (864)

Sol. $U_i = \frac{1}{2} \times 14 \times 12 \times 12 \text{ pJ} \quad (\because U = \frac{1}{2}CV^2)$

= 1008 pJ

$U_f = \frac{1008}{7} \text{ pJ} = 144 \text{ pJ} \quad (\because U = \frac{1}{2}CV^2)$

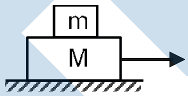
Mechanical energy = ΔU

= 1008 – 144

= 864 pJ

9. Two blocks (m = 0.5 kg and M = 4.5 kg) are arranged on a horizontal frictionless table as shown in figure. The coefficient of static friction between the two blocks is $\frac{3}{7}$. Then the maximum horizontal force that can be applied on the larger block so that the blocks move together is ______ N.

[Take g as 9.8 ms^{-2}]



Ans. (21)

Sol. $a_{max} = \mu g = \frac{3}{7} \times 9.8$

F = (M + m) a_{max} = 5 a_{max}

= 21 Newton

10. If 2.5×10^{-6} N average force is exerted by a light wave on a non-reflecting surface of 30 cm^2 area during 40 minutes of time span, the energy flux of light just before it falls on the surface is ____ W/cm^2.

(Assume complete absorption and normal incidence conditions are there)

Ans. (25)

Sol. $F = \frac{IA}{C}$

$I = \frac{FC}{A} = \frac{2.5 \times 10^{-6} \times 3 \times 10^8}{30} = 25 \text{ W / cm}^2$

PART B : CHEMISTRY

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. With respect to drug-enzyme interaction, identify the wrong statement:

(1) Non-Competitive inhibitor binds to the allosteric site

(2) Allosteric inhibitor changes the enzyme's active site

(3) Allosteric inhibitor competes with the enzyme's active site

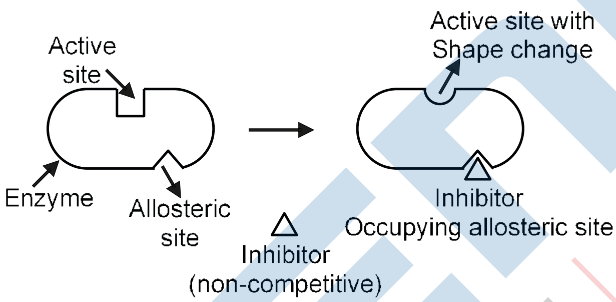
(4) Competitive inhibitor binds to the enzyme's active site

Ans. (3)

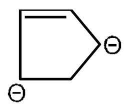
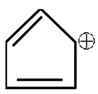
Sol. Some drug do not bind to the Enzyme's active site. These bind to a different site of enzyme which called allosteric site.

This binding of inhibitor at allosteric site changes the shape of the active site in such a way that substrate can not recognise it.

Such inhibitor is known as Non-competitive inhibitor.

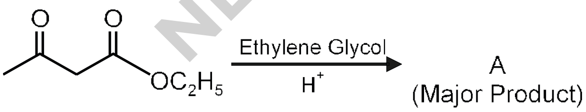


2. Which of the following is an aromatic compound?

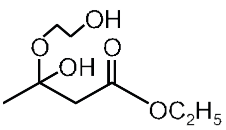
(1)  (2)  (3)  (4) 

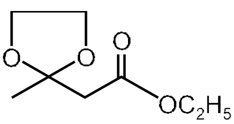
Ans. (1)

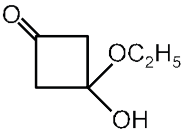
Sol.  → Aromatic compound

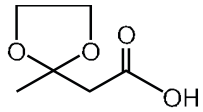
3. 

The product "A" in the above reaction is:

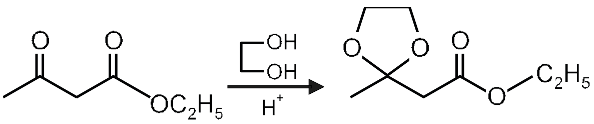
(1) 

(2) 

(3) 

(4) 

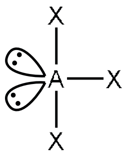
Ans. (2)

Sol. 

4. A central atom in a molecule has two lone pairs of electrons and forms three single bonds. The shape of this molecule is:

(1) see-saw (2) planar triangular (3) T-shaped (4) trigonal pyramidal

Ans. (3)

Sol. 

sp^3d hybridised

T-shaped

5. Given below are two statements:

Statement I : Potassium permanganate on heating at 573 K forms potassium manganate.

Statement II : Both potassium permanganate and potassium manganate are tetrahedral and paramagnetic in nature.

In the light of the above statements, choose the most appropriate answer from the options given below:

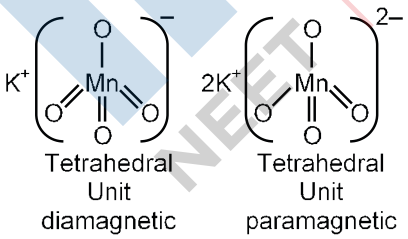
(1) Statement I is true but statement II is false (2) Both statement I and statement II are true

(3) Statement I is false but statement II is true (4) Both statement I and statement II are false

Ans. (1)

Sol. $2KMnO_4 \xrightarrow{573\ K} K_2MnO_4 + MnO_2 + O_2$

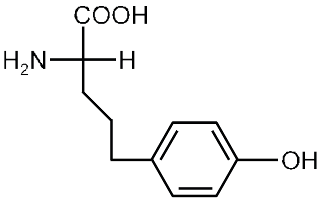
Potassium permanganate → Potassium manganate

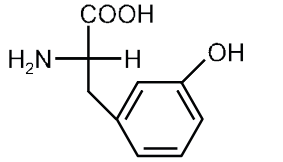


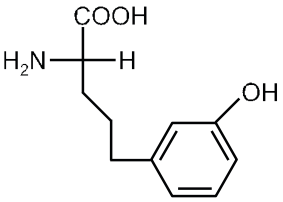
Statement-I is correct.

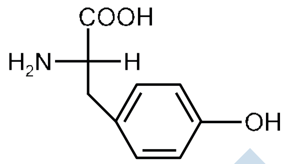
Statement-II is incorrect.

6. Which of the following is correct structure of tyrosine?

(1) 

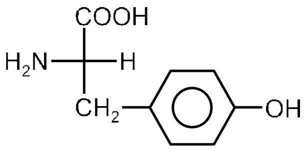
(2) 

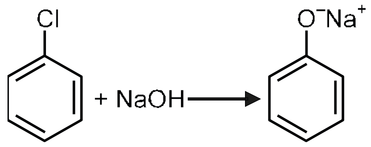
(3) 

(4) 

Ans. (4)

Sol. The structure of Tyrosine amino acid is



7. 

The above reaction requires which of the following reaction conditions?

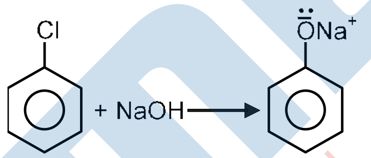
(1) 573 K, Cu, 300 atm

(2) 623 K, Cu, 300 atm

(3) 573 K, 300 atm

(4) 623 K, 300 atm

Ans. (4)

Sol. 

Dow process

Temperature = 623 K

Pressure = 300 atm

8. The absolute value of the electron gain enthalpy of halogens satisfies:

(1) I > Br > Cl > F (2) Cl > Br > F > I (3) Cl > F > Br > I (4) F > Cl > Br > I

Ans. (3)

Sol. Order of electron gain enthalpy

(Absolute value)

Cl > F > Br > I

9. Which of the following compound CANNOT act as a Lewis base?

(1) NF_3 (2) PCl_5 (3) SF_4 (4) ClF_3

Ans. (2)

Sol. Lewis base : Chemical species which has capability to donate electron pair.

In NF_3, SF_4, ClF_3 central atom (i.e. N, S, Cl) having lone pair therefore act as lewis base.

In PCl_5 central atom (P) does not have lone pair therefore does not act as lewis base.

10. Reducing smog is a mixture of:

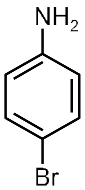
(1) Smoke, fog and O_3 (2) Smoke, fog and SO_2

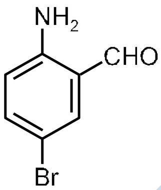
(3) Smoke, fog and CH_2=CH–CHO (4) Smoke, fog and N_2O_3

Ans. (2)

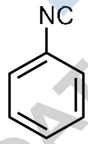
Sol. Reducing or classical smog is the combination of smoke, fog and SO_2.

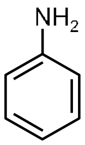
11. Hoffmann bromomide degradation of benzamide gives product A, which upon heating with $CHCl_3$ and NaOH gives product B. The structures of A and B are :

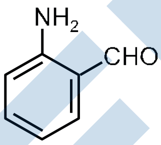
(1) A - 

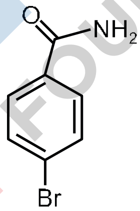
B - 

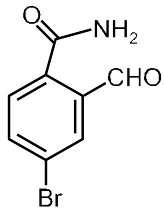
(2) A - 

B - 

(3) A - 

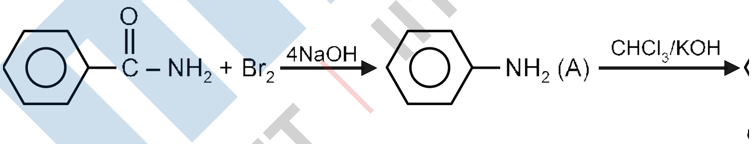
B - 

(4) A - 

B - 

Ans. (2)

Sol. Hoffmann bromamide degradation reaction :

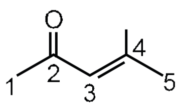


12. Mesityl oxide is a common name of :

(1) 2,4-Dimethyl pentan-3-one (2) 3-Methyl cyclohexane carbaldehyde

(3) 2-Methyl cyclohexanone (4) 4-Methyl pent-3-en-2-one

Ans. (4)

Sol. 

Mesityloxide

IUPAC [4-Methylpent-3-en-2-one]

13. Which of the following reaction is an example of ammonolysis?

(1) $C_6H_5COCl + C_6H_5NH_2 \longrightarrow C_6H_5CONHC_6H_5$

(2) $C_6H_5CH_2CN \xrightarrow{[H]} C_6H_5CH_2CH_2NH_2$

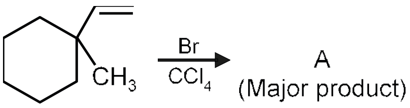
(3) $C_6H_5NH_2 \xrightarrow{HCl} C_6H_5\overset{+}{N}H_3Cl^-$

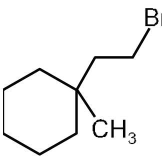
(4) $C_6H_5CH_2Cl + NH_3 \longrightarrow C_6H_5CH_2NH_2$

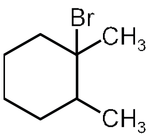
Ans. (4)

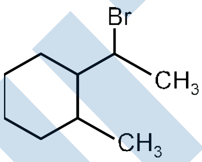
Sol. The process of cleavage of the C–X bond by Ammonia molecule is known as ammonolysis.

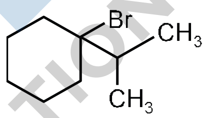
Ex : $E - CH_2 - Cl + \ddot{N}H_3 \longrightarrow R - CH_2 - NH_2$

14. 

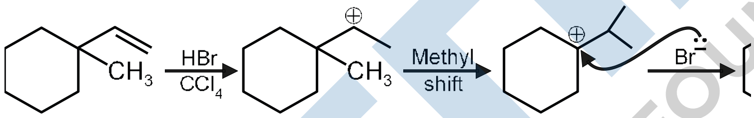
(1) 

(2) 

(3) 

(4) 

Ans. (4)

Sol. 

15. A colloidal system consisting of a gas dispersed in a solid is called a/an :

(1) solid sol (2) gel (3) aerosol (4) foam

Ans. (1)

Sol. Colloid of gas dispersed in solid is called solid sol.

16. The INCORRECT statement(s) about heavy water is (are)

(A)used as a moderator in nuclear reactor
(B) obtained as a by-product in fertilizer industry.
(C) used for the study of reaction mechanism
(D) has a higher dielectric constant than water

Choose the correct answer from the options given below :

(1) (B) only (2) (C) only (3) (D) only (4) (B) and (D) only

Ans. (3)

Sol. The dielectric constant of H_2O is greater than heavy water.

17. The correct order of conductivity of ions in water is :

(1) $Na^+ > K^+ > Rb^+ > Cs^+$
(2) $Cs^+ > Rb^+ > K^+ > Na^+$
(3) $K^+ > Na^+ > Cs^+ > Rb^+$
(4) $Rb^+ > Na^+ > K^+ > Li^+$

Ans. (2)

Sol. Li^+ Na^+ K^+ Rb^+ Cs^+ ⟶ Hydration energy ↑

⟶ Ionic mobility ↓

⟶ Conductivity ↓

$\therefore$ Correct option is $Na^+ > K^+ > Rb^+ > Cs^+$.

OR

Sol. As the size of gaseous ion decreases, it get more hydrated in water and hence, the size of aqueous ion increases. When this bulky ion move in solution, it experience greater resistance and hence lower conductivity.

Size of gaseous ion : $Cs^+ > Rb^+ > K^+ > Na^+$

Size of aqueous ion : $Cs^+ < Rb^+ < K^+ < Na^+$

Conductivity : $Cs^+ > Rb^+ > K^+ > Na^+$

18. What is the spin-only magnetic moment value (BM) of a divalent metal ion with atomic number 25, in it's aqueous solution?

(1) 5.92 (2) 5.0 (3) zero (4) 5.26

Ans. (1)

Sol. Electronic configuration of divalent metal ion having atomic number 25 is

$Mn^{+2}_{(aq)} \Rightarrow$ $1s^2$ [⥮] $2s^2$ [⥮] $2p^6$ [⥮|⥮|⥮] $3s^2$ [⥮] $3p^6$ [⥮|⥮|⥮] $3d^5$ [↿|↿|↿|↿|↿]

Total number of unpaired electrons = 5

μ (Magentic moment) $= \sqrt{n(n+2)}$ BM

where n = number of unpaired e^-

$\therefore \mu = \sqrt{5(5+2)} = \sqrt{35}$ BM $= 5.92$ BM

19. Given below are two statements :

Statement-I : Retardation factor (R_f) can be measured in meter/centimeter.

Statement-II : R_f value of a compound remains constant in all solvents.

Choose the most appropriate answer from the options given below:

(1) Statement-I is true but statement-II is false (2) Both statement-I and statement-II are true

(3) Both statement-I and statement-II are false (4) Statement-I is false but statement-II is true

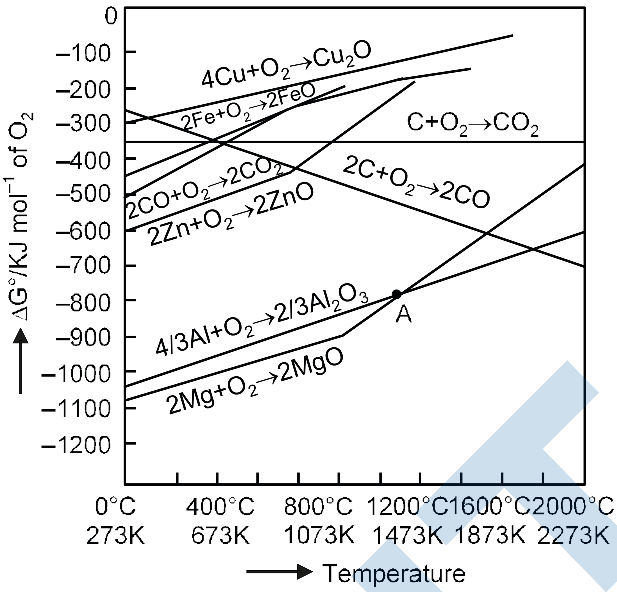
Ans. (3)

Sol. R_f = retardation factor

$$R_f = \frac{\text{Distance travelled by the substance form reference line (c.m)}}{\text{Distance travelled by the solvent form reference line (c.m)}}$$

Note : R_f value of different compounds are different.

20. The point of intersection and sudden increase in the slope, in the diagram given below, respectively, indicates :



(1) $\Delta G = 0$ and melting or boiling point of the metal oxide

(2) $\Delta G > 0$ and decomposition of the metal oxide

(3) $\Delta G < 0$ and decomposition of the metal oxide

(4) $\Delta G = 0$ and reduction of the metal oxide

Ans. (1)

ZIGYAN Ans. (Bonus)

Sol. At intersection point $\Delta G = 0$ and sudden increase in slope is due to melting or boiling point of the metal.

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. The reaction of white phosphorus on boiling with alkali in inert atmosphere resulted in the formation of product 'A'. The reaction 1 mol of 'A' with excess of $AgNO_3$ in aqueous medium gives ________ mol(s) of Ag.

Ans. (4)

Sol. $P_4 + 3OH^- + 3H_2O \longrightarrow PH_3 + \underset{(A)}{3H_2PO_2^-}$

$\underset{1\text{ mole}}{H_2PO_2^-} + \underset{\text{excess}}{4Ag^+} + 2H_2O \longrightarrow \underset{4\text{ mole}}{4Ag} + H_3PO_4 + 3H^+$

2. 0.01 moles of a weak acid HA ($K_a = 2.0 \times 10^{-6}$) is dissolved in 1.0 L of 0.1 M HCl solution. The degree of dissociation of HA is ________ $\times 10^{-5}$.

[Neglect volume change on adding HA. Assume degree of dissociation <<1]

Ans. (2)

Sol.

	HA	⇌	H^+	+	A^-
Initial conc.	0.01M		0.1M		0
Equ. conc.	(0.01 – x)		(0.1 + x)		xM
	≈ 0.01M		≈ 0.1M		

Now, $K_a = \frac{[x^+][A^-]}{[HA]} \Rightarrow 2 \times 10^{-6} = \frac{0.1 \times x}{0.01}$

$\therefore$ x = 2 × 10^{-7}

Now, $\alpha = \frac{x}{0.01} = \frac{2 \times 10^{-7}}{0.01} = 2 \times 10^{-5}$

3. A certain orbital has n = 4 and m_L = –3. The number of radial nodes in this orbital is ________.

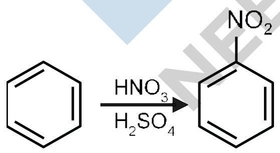
Ans. (0)

Sol. n = 4 and m_ℓ = –3

Hence, ℓ value must be 3.

Now, number of radial nodes = n – ℓ – 1

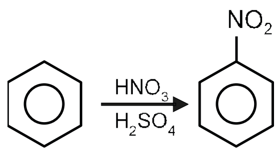
= 4 – 3 – 1 = 0

4. 

In the above reaction, 3.9 g of benzene on nitration gives 4.92 g of nitrobenzene. The percentage yield of nitrobenzene in the above reaction is _______ %.

(Given atomic mass : C : 12.0 u, H : 1.0u, O : 16.0 u, N : 14.0 u)

Ans. (80)

Sol. 

1 mole	1 mole
78gm	123gm
3.9gm	$\frac{123}{78} \times 3.9 = 6.15$ gm

But actual amount of nitrobenzene formed is 4.92 gm and hence.

$$\text{Percentage yield} = \frac{4.92}{6.15} \times 100 = 80\%$$

5. The mole fraction of a solute in a 100 molal aqueous solution _______ × 10^{-2}.

[Given : Atomic masses : H : 1.0 u, O : 16.0 u]

Ans. (64)

Sol. 100 molal aqueous solution means there is 100 mole solute in 1 kg = 1000 gm water.

Now,

$$\text{mole} - \text{fraction of solute} = \frac{n_{solute}}{n_{solute} + n_{solvent}} = \frac{100}{100 + \frac{1000}{18}} = \frac{1800}{2800} = 0.6428$$

= 64.28×10^{-2}

6. For a certain first order reaction 32% of the reactant is left after 570 s. The rate constant of this reaction is _______ × 10^{-3} s^{-1}. [Given : $\log_{10}2 = 0.301$, ln10 = 2.303]

Ans. (2)

Sol. For 1st order reaction,

$$K = \frac{2.303}{t} \cdot \log\frac{[A_0]}{[A_t]} = \frac{2.303}{570\text{ sec}} \cdot \log\left(\frac{100}{32}\right)$$

= 1.999×10^{-3} $sec^{-1} \approx 2 \times 10^{-3}$ sec^{-1}

7. The standard enthalpies of formation of Al_2O_3 and CaO are –1675 kJ mol^{-1} and –635 kJ mol^{-1} respectively. For the reaction

$3CaO + 2Al \rightarrow 3Ca + Al_2O_3$ the standard reaction enthalpy Δ_rH^0 = _______ kJ.

Ans. (230)

Sol. Given reaction:

$3CaO + Al \rightarrow Al_2O_3 + 3Ca$

Now, $\Delta_rH° = S\,\Delta_fH°_{Products} - S\Delta_fH°_{Reactants}$

= [1 × (–1675) + 3 × 0] – [3 × (–635) + 2 × 0]

= + 230 kJ mol^{-1}

8. 15 mL of aqueous solution of Fe^{2+} in acidic medium completely reacted with 20 mL of 0.03 M aqueous $Cr_2O_7^{2-}$. The molarity of the Fe^{2+} solution is _______ × 10^{-2} M.

Ans. (24)

Sol. $n_{eq}\ Fe^{2+} = n_{eq}\ Cr_2O_7^{2-}$ or, $\left(\frac{15 \times M_{Fe^{2+}}}{1000}\right) \times 1 = \left(\frac{20 \times 0.03}{1000}\right) \times 6$

$\therefore M_{Fe^{2+}} = 0.24\ M = 24 \times 10^{-2}\ M$

9. The oxygen dissolved in water exerts a partial pressure of 20 kPa in the vapour above water. The molar solubility of oxygen in water is _______ × 10^{-5} mol dm^{-3}.

[Given : Henry's law constant = K_H = 8.0 × 10^4 kPa for O_2.

Density of water with dissolved oxygen = 1.0 kg dm^{-3}]

Ans. (25)

ZIGYAN Ans. (1389)

Sol. $P = K_H \cdot x$

or, $20 \times 10^3 = (8 \times 10^4 \times 10^3) \times \frac{n_{O_2}}{n_{O_2} + n_{water}}$

or, $\frac{1}{4000} = \frac{n_{O_2}}{n_{O_2} + n_{water}} = \frac{n_{O_2}}{n_{water}}$

Means 1 mole water (= 18 gm = 18 ml) dissolves

$\frac{1}{4000}$ moles O_2. Hence, molar solubility

$= \frac{\left(\frac{1}{4000}\right)}{18} \times 1000 = \frac{1}{72}$ mol dm^{-3}

= 1388.89 × 10^{-5} mol dm^{-3} ≈ 1389 mol dm^{-3}

10. The pressure exerted by a non-reactive gaseous mixture of 6.4 g of methane and 8.8 g of carbon dioxide in a 10 L vessel at 27°C is _______ kPa.

[Assume gases are ideal, R = 8.314 J mol^{-1} K^{-1}

Atomic masses : C : 12.0 u, H : 1.0 u, O : 16.0 u]

Ans. (150)

Sol. Total moles of gases, $n = n_{CH_4} + n_{CO_2} = \frac{6.4}{16} + \frac{8.8}{44} = 0.6$

Now, $P = \frac{nRT}{V} = \frac{0.6 \times 8.314 \times 300}{10 \times 10^{-3}}$

= 1.49652 × 10^5 Pa = 149.652 kPa

≈ 150 kPa

PART C : MATHEMATICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. The inverse of $y = 5^{\log x}$ is :

(1) $x = 5^{\log y}$ (2) $x = y^{\log 5}$ (3) $x = y^{\frac{1}{\log 5}}$ (4) $x = 5^{\frac{1}{\log y}}$

Ans. (3)

ZIGYAN Ans. (1 or 2 or 3)

Sol. $y = 5^{\log x}$

$y = x^{\log 5}$

$y^{\frac{1}{\log x}} = x$

Replying $x \to y$ and $y \to x$

2. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 7\hat{i} + \hat{j} - 6\hat{k}$.

If $\vec{r} \times \vec{a} = \vec{r} \times \vec{b}, \vec{r} \cdot \left(\hat{i} + 2\hat{j} + \hat{k}\right) = -3$, then $\vec{r} \cdot \left(2\hat{i} - 3\hat{j} + \hat{k}\right)$ is equal to :

(1) 12 (2) 8 (3) 13 (4) 10

Ans. (1)

Sol. $\vec{r} \times \vec{a} - \vec{r} \times \vec{b} = 0$

$\Rightarrow \vec{r} \times (\vec{a} - \vec{b}) = 0$

$\Rightarrow \vec{r} = \lambda(\vec{a} - \vec{b})$

$\Rightarrow \vec{r} = \lambda(-5\hat{i} - 4\hat{j} + 10\hat{k})$

Also $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -3$

$\Rightarrow \lambda(-5 - 8 + 10) = -3$

$\lambda = 1$

Now $\vec{r} = -5\hat{i} - 4\hat{j} + 10\hat{k}$

$= \vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k})$

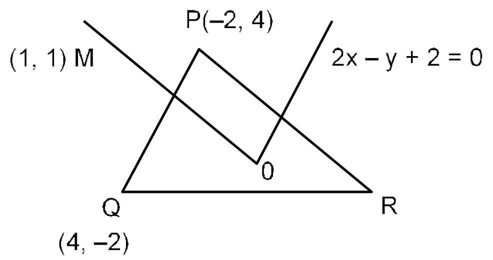
$= -10 + 12 + 10 = 12$

3. In a triangle PQR, the co-ordinates of the points P and Q are (–2, 4) and (4, –2) respectively. If the equation of the perpendicular bisector of PR is $2x - y + 2 = 0$, then the centre of the circumcircle of the ΔPQR is :

(1) (–1, 0) (2) (–2, –2) (3) (0, 2) (4) (1, 4)

Ans. (2)

Sol.



Equation of perpendicular bisector of PR is y = x

Solving with 2x – y + 2 = 0 will give (–2, 2)

4. The system of equations kx + y + z = 1, x + ky + z = k and x + y + zk = k^2 has no solution if k is equal to:

(1) 0 (2) 1 (3) –1 (4) –2

Ans. (4)

Sol. kx + y + z = 1

x + ky + z = k

x + y + zk = k^2

$$\Delta = \begin{vmatrix} K & 1 & 1 \\ 1 & K & 1 \\ 1 & 1 & K \end{vmatrix} = K(K^2 - 1) - 1(k-1) + 1(1-K)$$

$= K^3 - K - K + 1 + 1 - K$

$= K^3 - 3K + 2$

$= (K-1)^2 (K+2)$

For K = 1

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

But for K = – 2, at least one out of $\Delta_1, \Delta_2, \Delta_3$ are not zero

Hence for no solution, K = –2

5. If $\cot^{-1}(\alpha) = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + ...$ upto 100 terms, then α is :

(1) 1.01 (2) 1.00 (3) 1.02 (4) 1.03

Ans. (1)

Sol. $\text{Cot}^{-1}(\alpha) = \cot^{-1}(2) + \cot^{-1}(8) + \cot^{-1}(18) + ...$

$$= \sum_{n=1}^{100} \tan^{-1}\left(\frac{2}{4n^2}\right)$$

$$= \sum_{n=1}^{100} \tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$$

$$= \sum_{n=1}^{100} \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$$

$= \tan^{-1} 201 - \tan^{-1} 1$

$= \tan^{-1}\left(\frac{200}{202}\right)$

$\therefore \cot^{-1}(\alpha) = \cot^{-1}\left(\frac{202}{200}\right)$

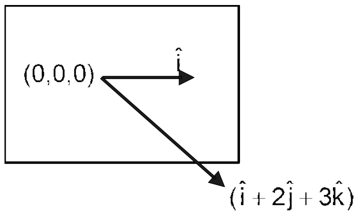
$\alpha = 1.01$

6. The equation of the plane which contains the y-axis and passes through the point (1, 2, 3) is :

(1) $x + 3z = 10$ (2) $x + 3z = 0$ (3) $3x + z = 6$ (4) $3x - z = 0$

Ans. (4)

Sol.



$\vec{n} = \hat{j} \times (\hat{i} + 2\hat{j} + 3\hat{k})$

$n = -3\hat{i} + 0\hat{j} + \hat{k}$

So, $(-3)(x-1) + 0(y-2) + (1)(z-3) = 0$

$\Rightarrow -3x + z = 0$

Alternate :

Required plane is

$$\begin{vmatrix} x & y & z \\ 0 & 1 & 0 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow 3x - z = 0$

7. If $A = \begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}$ and $\det\left(A^2 - \frac{1}{2}I\right) = 0$ then a possible value of α is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

Ans. (3)

Sol. $A^2 = \sin^2\alpha\, I$

So, $\left|A^2 - \frac{1}{2}\right| = \left(\sin^2\alpha - \frac{1}{2}\right)^2 = 0$

$\Rightarrow \sin\alpha = \pm\frac{1}{\sqrt{2}}$

8. If the Boolean expression $(p \Rightarrow q) \Leftrightarrow (q * (\sim p))$ is a tautology, then the Boolean expression $p * (\sim q)$ is equivalent to :

(1) $q \Rightarrow p$ (2) $\sim q \Rightarrow p$ (3) $p \Rightarrow \sim q$ 4) $p \Rightarrow q$

Ans. (1)

Sol. $\because p \rightarrow q \equiv \sim p \vee q$

So, $* \equiv v$

Thus, $p * (\sim q) \equiv p \vee (\sim q)$

$\equiv q \rightarrow p$

9. Two dices are rolled. If both dices have six faces numbered 1,2,3,5,7 and 11, then the probability that the sum of the numbers on the top faces is less than or equal to 8 is :

(1) $\frac{4}{9}$ (2) $\frac{17}{36}$ (3) $\frac{5}{12}$ (4) $\frac{1}{2}$

Ans. (2)

Sol. n(E) = 5 + 4 + 4 + 3 + 1 = 17

So, $P(E) = \frac{17}{36}$

10. If the fourth term in the expansion of $(x + x^{\log_2 x})^7$ is 4480, then the value of x where $x \in N$ is equal to :

(1) 2 (2) 4 (3) 3 (4) 1

Ans. (1)

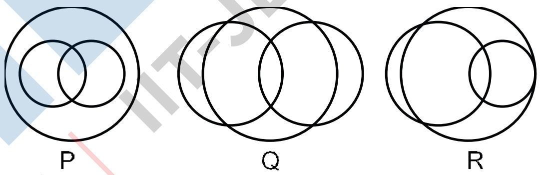
Sol. ${}^7C_3 x^4\ x^{(3\log_2 x)} = 4480$

$\Rightarrow x^{(4+3\log_2 x)} = 2^7$

$\Rightarrow (4 + 3t)t = 7;\ t = \log_2 x$

$\Rightarrow t = 1, \frac{-7}{3} \Rightarrow x = 2$

11. In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statement?



(1) P and Q (2) P and R (3) None of these (4) Q and R

Ans. (3)

Sol. $A \cap B \cap C$ is visible in all three venn diagram

Hence, Option (3)

12. The sum of possible values of x for $\tan^{-1}(x+1) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1}\left(\frac{8}{31}\right)$ is :

(1) $-\frac{32}{4}$ (2) $-\frac{31}{4}$ (3) $-\frac{30}{4}$ (4) $-\frac{33}{4}$

Ans. (1)

Sol. $\tan^{-1}(x+1)+\cot^{-1}\left(\frac{1}{x-1}\right)=\tan^{-1}\frac{8}{31}$

Taking tangent both sides :-

$\frac{(x+1)+(x-1)}{1-(x^2-1)}=\frac{8}{31}$

$\Rightarrow \frac{2x}{2-x^2}=\frac{8}{31}$

$\Rightarrow 4x^2 + 31x - 8 = 0$

$\Rightarrow x = -8, \frac{1}{4}$

But, if $x=\frac{1}{4}$

$\tan^{-1}(x+1)\in\left(0,\frac{\pi}{2}\right)$

$\&\cot^{-1}\left(\frac{1}{x-1}\right)\in\left(\frac{\pi}{2},\pi\right)$

$\Rightarrow \text{LHS} > \frac{\pi}{2} \;\&\; \text{RHS} < \frac{\pi}{2}$

(Not possible)

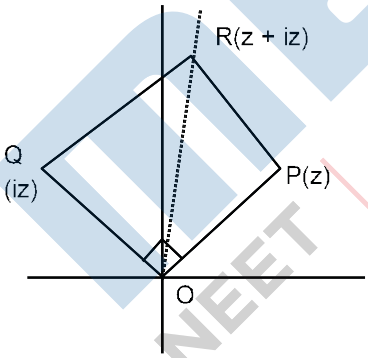
Hence, x = – 8

13. The area of the triangle with vertices A(z), B(iz) and C (z + iz) is :

(1) 1 (2) $\frac{1}{2}|z|^2$ (3) $\frac{1}{2}$ (4) $\frac{1}{2}|z+iz|^2$

Ans. (2)

Sol.



$A=\frac{1}{2}|z||iz|$

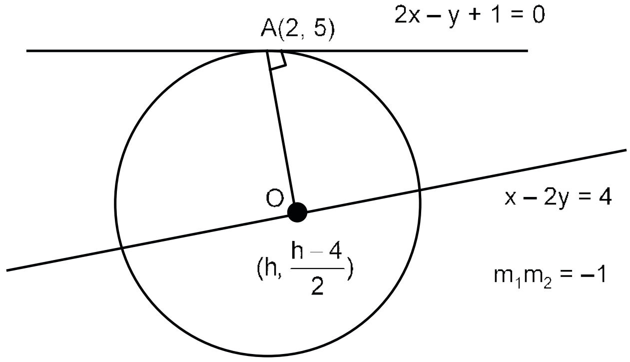
$=\frac{|z|^2}{2}$

14. The line $2x - y + 1 = 0$ is a tangent to the circle at the point (2, 5) and the centre of the circle lies on $x - 2y = 4$. Then, the radius of the circle is:

(1) $3\sqrt{5}$ (2) $5\sqrt{3}$ (3) $5\sqrt{4}$ (4) $4\sqrt{5}$

Ans. (1)

Sol.



$$\left(\frac{h - \frac{h-4}{2}}{2-h}\right)(2) = -1$$

$h = 8$

center (8, 2)

$$\text{Radius} = \sqrt{(8-2)^2 + (2-5)^2} = 3\sqrt{5}$$

15. Team 'A' consists of 7 boys and n girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then n is equal to :

(1) 5 (2) 2 (3) 4 (4) 6

Ans. (3)

Sol. Total matches between boys of both team = ${}^7C_1 \times {}^4C_1 = 28$

Total matches between girls of both team = ${}^nC_1\ {}^6C_1 = 6n$

Now, $28 + 6n = 52$

$\Rightarrow n = 4$

16. The value of $4 + \cfrac{1}{5 + \cfrac{1}{4 + \cfrac{1}{5 + \cfrac{1}{4 + \ldots\ldots \infty}}}}$ is :

(1) $2 + \frac{2}{5}\sqrt{30}$ (2) $2 + \frac{4}{\sqrt{5}}\sqrt{30}$ (3) $4 + \frac{4}{\sqrt{5}}\sqrt{30}$ (4) $5 + \frac{2}{5}\sqrt{30}$

Ans. (1)

Sol. $y = 4 + \frac{1}{\left(5 + \frac{1}{y}\right)}$

$y - 4 = \frac{1}{(5y + 1)}$

$5y^2 - 20y - 4 = 0$

$y = \frac{20 + \sqrt{480}}{10}$

$y = \frac{20 - \sqrt{480}}{10} \rightarrow$ rejected

$y = 2 + \sqrt{\frac{480}{100}}$ Correct with Option (1)

17. Choose the incorrect statement about the two circles whose equations are given below :

$x^2 + y^2 - 10x - 10y + 41 = 0$ and

$x^2 + y^2 - 16x - 10y + 80 = 0$

(1) Distance between two centres is the average of radii of both the circles.

(2) Both circles' centres lie inside region of one another.

(3) Both circles pass through the centre of each other.

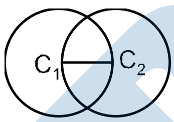
4) Circles have two intersection points.

Ans. (2)

Sol. $r_1 = 3, c_1\ (5, 5)$

$r_2 = 3, c_2\ (8, 5)$

$C_1C_2 = 3, r_1 = 3, r_2 = 3$



18. Which of the following statements is incorrect for the function $g(\alpha)$ for $\alpha \in R$ such that

$$g(\alpha) = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin^\alpha x}{\cos^\alpha x + \sin^\alpha x} dx$$

(1) $g(\alpha)$ is a strictly increasing function

(2) $g(\alpha)$ has an inflection point at $\alpha = \frac{1}{2}$

(3) $g(\alpha)$ is a strictly decreasing function

(4) $g(\alpha)$ is an even function

Ans. (4)

ZIGYAN Ans. (1 or 2 or 3/Bonus)

Sol. $g(\alpha)=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin^{\alpha}x}{(\sin^{\alpha}x+\cos^{\alpha}x)}$ .....(i)

$g(\alpha)=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\cos^{\alpha}x}{(\sin^{\alpha}x+\cos^{\alpha}x)}$ .....(ii)

(1) + (2)

$2g(\alpha)=\frac{\pi}{6}$

$g(\alpha)=\frac{\pi}{12}$

Constant and even function

Due to typing mistake it must be bonus.

19. Which of the following is true for y(x) that satisfies the differential equation

$\frac{dy}{dx}=xy-1+x-y\ ;\ y(0)=0$:

(1) $y(1)=e^{-\frac{1}{2}}-1$ (2) $y(1)=e^{\frac{1}{2}}-e^{-\frac{1}{2}}$ (3) y(1) = 1 (4) $y(1)=e^{\frac{1}{2}}-1$

Ans. (1)

Sol. $\frac{dy}{dx}=(1+y)(x-1)$

$\frac{dy}{(y+1)}=(x-1)dx$

Integrate $\ln(y+1)=\frac{x^2}{2}-x+c$

$(0,0)\Rightarrow c=0\Rightarrow y=e^{\left(\frac{x^2}{2}-x\right)}-1$

20. The value of $\lim_{x\to0^+}\frac{\cos^{-1}(x-[x]^2)\cdot\sin^{-1}(x-[x]^2)}{x-x^3}$, where

[x] denotes the greatest integer $\le$ x is :

(1) π (2) 0 (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

Ans. (4)

Sol. $\lim_{x\to0^+}\frac{\cos^{-1}x}{(1-x^2)}\times\frac{\sin^{-1}x}{x}=\frac{\pi}{2}$

Numeric Value Type

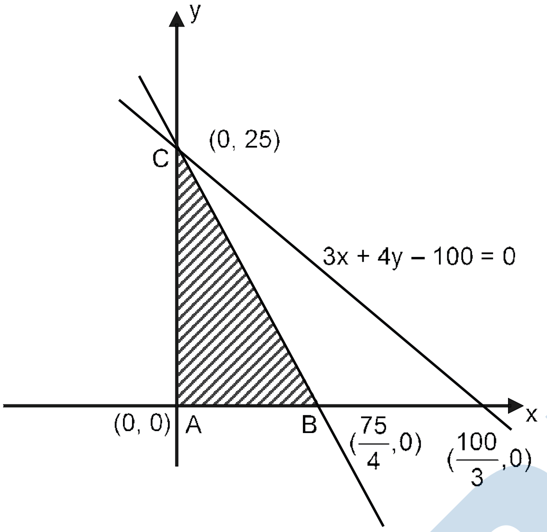
This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. The maximum value of z in the following equation $z = 6xy + y^2$, where $3x + 4y \le 100$ and $4x + 3y \le 75$ for $x \ge 0$ and $y \ge 0$ is _______ .

Ans. (904)

ZIGYAN Ans. (904 or 904.01 or 904.02)

Sol.



$z = 6xy + y^2 = y(6x + y)$

$3x + 4y \le 100$ .....(i)

$4x + 3y \le 75$ ......(ii)

$x \ge 0$

$y \ge 0$

$x \le \frac{75-3y}{4}$

$Z = y(6x + y)$

$Z \le y\left(6\cdot\left(\frac{75-3y}{4}\right)+y\right)$

$Z \le \frac{1}{2}(225y-7y^2) \le \frac{(225)^2}{2\times4\times7}$

$= \frac{50625}{56}$

≈ 904.0178

≈ 904.02

It will be attained at $y = \frac{225}{14}$

2. If the function $f(x)=\frac{\cos(\sin x)-\cos x}{x^4}$ is continuous at each point in its domain and $f(0)=\frac{1}{k}$, then k is ________ .

Ans. (6)

Sol. $\lim\limits_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}=f(0)$

$\Rightarrow \lim\limits_{x\to 0}\frac{2\sin\left(\frac{\sin x+x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4}=\frac{1}{K}$

$\Rightarrow \lim\limits_{x\to 0}2\left(\frac{\sin x+x}{2x}\right)\left(\frac{x-\sin x}{2x^3}\right)=\frac{1}{K}$

$\Rightarrow 2\times\frac{(1+1)}{2}\times\frac{1}{2}\times\frac{1}{6}=\frac{1}{K}$

$\Rightarrow$ K = 6

3. If $f(x)=\sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$ and its first derivative with respect to x is $-\frac{b}{a}\log_e 2$ when x = 1, where a and b are integers, then the minimum value of $|a^2-b^2|$ is _______ .

Ans. (481)

Sol. $f(x)=\sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$ at x= 1 ; 2^{2x} = 4

for $\sin\left(\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right)$;

Let tan–1 x = θ ; $\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\therefore$ $\sin(\cos^{-1}\cos 2\theta)=\sin 2\theta$

$$\left\{\begin{matrix}\text{If} & x>1\Rightarrow\frac{\pi}{2}>\theta>\frac{\pi}{4}\\ \therefore & \pi>2\theta>\frac{\pi}{2}\end{matrix}\right\}$$

$=2\sin\theta\cos\theta=\frac{2\tan\theta}{1+\tan^2\theta}$

$=\frac{2x}{1+x^2}$

Hence, $f(x)=\frac{2\cdot 2^x}{1+2^{2x}}$

$\therefore f'(x)=\frac{(1+2^{2x})(2\cdot 2^x\ln 2)-2^{2x}\cdot 2\cdot\ln 2\cdot 2\cdot 2^x}{(1+2^{2x})}$

$\therefore f^1(1) = \frac{20\ln 2 - 32\ln 2}{25} = -\frac{12}{25}\ln 2$

So, a = 25, b = 12 ⇒ $|a^2 - b^2| = 25^2 - 12^2$

$= 625 - 144$

$= 481$

4. Let there be three independent events E_1, E_2 and E_3. The probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let 'p' denote the probability of none of events occurs that satisfies the equations $(\alpha - 2\beta)\, p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then, $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ to _____.

Ans. (6)

Sol. Let $P(E_1) = P_1$; $P(E_2) = P_2$; $P(E_3) = P_3$

$P(E_1 \cap \bar{E}_2 \cap \bar{E}_3) = \alpha = P_1(1-P_2)(1-P_3)$ ......(1)

$P(\bar{E}_1 \cap E_2 \cap \bar{E}_3) = \beta = (1-P_1)P_2(1-P_3)$ ......(2)

$P(\bar{E}_1 \cap \bar{E}_2 \cap E_3) = \gamma = (1-P_1)(1-P_2)P_3$ ......(3)

$P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3) = P = (1-P_1)(1-P_2)(1-P_3)$ ......(4)

Given that, $(\alpha - 2\beta)\, P = \alpha\beta$

$\Rightarrow (P_1(1 - P_2)(1 - P_3) - 2(1 - P_1)P_2(1 - P_3))P = P_1P_2$

$(1 - P_1)(1 - P_2)(1 - P_3)^2$

$\Rightarrow (P_1(1 - P_2) - 2(1 - P_1)P_2) = P_1P_2$

$\Rightarrow (P_1 - P_1P_2 - 2P_2 + 2P_1P_2) = P_1P_2$

$\Rightarrow P_1 = 2P_2$ ......(1)

and similarly, $(\beta - 3\gamma)P = 2B\gamma$

$P_2 = 3P_3$ ......(2)

So, $P_1 = 6P_3 \Rightarrow \boxed{\frac{P_1}{P_3} = 6}$

5. If $\vec{a} = \alpha\hat{i} + \beta\hat{j} + 3\hat{k}$, $\vec{b} = -\beta\hat{i} - \alpha\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} - \hat{k}$ such that $\vec{a}\cdot\vec{b} = 1$ and $\vec{b}\cdot\vec{c} = -3$, then $\frac{1}{3}\left(\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}\right)$ is equal to _______.

Ans. (2)

Sol. $\vec{a}\cdot\vec{b} = 1 \Rightarrow \quad -\alpha\beta - \alpha\beta - 3 = 1$

$\Rightarrow -2\alpha\beta = 4 \Rightarrow \boxed{\alpha\beta = -2}$ ....(1)

$\vec{b}\cdot\vec{c} = -3 \Rightarrow \quad -\beta + 2\alpha + 1 = -3$

$\boxed{\beta - 2\alpha = 4}$ ....(2)

Solving (1) & (2), $(\alpha, \beta) = (-1, 2)$

$$\frac{1}{3}[\vec{a}\,\vec{b}\,\vec{c}] = \frac{1}{3}\begin{vmatrix} \alpha & \beta & 3 \\ -\beta & -\alpha & -1 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= \frac{1}{3}\begin{vmatrix} -1 & 2 & 3 \\ -2 & 1 & -1 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= \frac{1}{3}\begin{vmatrix} 0 & 0 & 2 \\ -2 & 1 & -1 \\ 1 & -2 & -1 \end{vmatrix} = \frac{1}{3}[2(4-1)] = 2$$

6. If $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, then the value of $\det(A^4) + \det(A^{10} - (\text{Adj}(2A))^{10})$ is equal to ________ .

Ans. (16)

Sol. $2A\ \text{adj}(2A) = |2A|I$

$\Rightarrow A\ \text{adj}(2A) = -4I$ ....(i)

Now, $E = |A^4| + |A^{10} - (\text{adj}(2A))^{10}|$

$$= (-2)^4 + \frac{|A^{20} - A^{10}(\text{adj } 2A)^{10}|}{|A|^{10}}$$

$$= 16 + \frac{|A^{20} - (A\ \text{adj}(2A))^{10}|}{|A|^{10}}$$

$$= 16 + \frac{|A^{20} - 2^{10}I|}{|A|^{10}} \qquad \text{(from (1))}$$

Now, characteristic roots of A are 2 and –1.

So, characteristic roots of A^{20} are 2^{10} and 1.

Hence, $(A^{20} - 2^{10}I)(A^{20} - I) = 0$

$\Rightarrow |A^{20} - 2^{10}I| = 0$ (as $A^{20} \neq I$)

$\Rightarrow E = 16$ Ans.

7. If [·] represents the greatest integer function, then the value of $\left|\int_0^{\sqrt{\frac{\pi}{2}}} \left[\left[x^2\right] - \cos x\right]dx\right|$ is ________ .

Ans. (1)

Sol. $I = \int_0^{\sqrt{\pi/2}} \left([x^2] + [-\cos x]\right)dx$

$$= \int_0^1 0\,dx + \int_0^{\sqrt{\pi/2}} dx + \int_0^{\sqrt{\pi/2}} (-1)\,dx$$

$$= \sqrt{\frac{\pi}{2}} - 1 - \sqrt{\frac{\pi}{2}} = -1$$

$\Rightarrow |I| = 1$

8. The minimum distance between any two points P_1 and P_2 while considering point P_1 on one circle and point P_2 on the other circle for the given circles' equations

$x^2 + y^2 - 10x - 10y + 41 = 0$

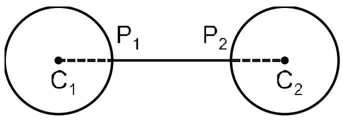
$x^2 + y^2 - 24x - 10y + 160 = 0$ is ______ .

Ans. (1)

Sol. Given $C_1(5, 5)$, $r_1 = 3$ and C_2 (12, 5), $r_2 = 3$

Now, $C_1C_2 > r_1 + r_2$

Thus, $(P_1P_2)_{min} = 7 - 6 = 1$



9. If the equation of the plane passing through the line of intersection of the planes $2x - 7y + 4z - 3 = 0$, $3x - 5y + 4z + 11 = 0$ and the point (–2, 1, 3) is $ax + by + cz - 7 = 0$, then the value of $2a + b + c - 7$ is _____ .

Ans. (4)

Sol. Required plane is

$p_1 + \lambda p_2 = (2 + 3\lambda)\,x - (7 + 5\lambda)\,y + (4 + 4\lambda)z - 3 + 11\lambda = 0$;

which is satisfied by (–2, 1, 3).

Hence, $\lambda = \frac{1}{6}$

Thus, plane is $15x - 47y + 28z - 7 = 0$

So, $2a + b + c - 7 = 4$

10. If $(2021)^{3762}$ is divided by 17, then the remainder is _______ .

Ans. (4)

Sol. $(2023 - 2)^{3762} = 2023k_1 + 2^{3762}$

$= 17k_2 + 2^{3762}$ (as $2023 = 17 \times 17 \times 9$)

$= 17k_2 + 4 \times 16^{940}$

$= 17k_2 + 4 \times (17 - 1)^{940}$

$= 17k_2 + 4\,(17k_3 + 1)$

$= 17k + 4 \Rightarrow$ remainder = 4